Density Operator and Entropy of the Damped Quantum Harmonic Oscillator

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Abstract. The expression for the density operator of the damped harmonic oscillator is derived from the master equation in the framework of the Lindblad theory for open quantum systems. Then the von Neumann entropy and effective temperature of the system are obtained. The entropy for a state characterized by a Wigner distribution function which is Gaussian in form is found to depend only on the variance of the distribution function.

1 Introduction

In the last two decades, the problem of dissipation in quantum mechanics, i.e. the consistent description of open quantum systems, was investigated by various authors [1-5] (for a recent review see ref. [6]). It is commonly understood [4, 7] that dissipation in an open system results from microscopic reversible interactions between the observable system and the environment. Because dissipative processes imply irreversibility and, therefore, a preferred direction in time, it is generally thought that quantum dynamical semigroups are the basic tools to introduce dissipation in quantum mechanics. In the Markov approximation the most general form of the generators of such semigroups was given by Lindblad [8]. This formalism has been studied for the case of damped harmonic oscillators [9, 10] and applied to various physical phenomena, for instance, the damping of collective modes in deep inelastic collisions in nuclear physics [11]. In [12] the Lindblad master equation for the harmonic oscillator was transformed into Fokker-Planck equations for quasiprobability distributions and a comparative study was made for the

Glauber P, antinormal ordering Q and Wigner W representations. In [13] the density matrix for the coherent state representation and the Wigner distribution function subject to different types of initial conditions were obtained for the damped harmonic oscillator. From a preceding analysis [14] resulted an analogy in form of the Lindblad master and Fokker-Planck equations with the corresponding equations from quantum optics, which is based on the quantum mechanics of the damped harmonic oscillator [4] and the corresponding Brownian motion. The equation of motion generally used in the theory of Brownian motion is the master equation for the density operator or the Fokker-Planck equation satisfied by the distribution function [4, 15, 16].

In the present study we are also concerned with the observable system of a harmonic oscillator which interacts with an environment. The aim of this work is to explore further the physical aspects of the Fokker-Planck equation which is the c-number equivalent equation to the master equation. Generally the master equation for the density operator gains considerably in clarity if it is represented in terms of the Wigner distribution function which satisfies the Fokker-Planck equation. Within the Lindblad theory for open quantum systems, we will describe the evolution of the considered system towards a final equilibrium state. First we derive a closed form of the density operator satisfying the Lindblad master equation and then calculate the von Neumann entropy and the effective temperature of the quantum-mechanical system in a state characterized by a Wigner distribution function which is Gaussian in form. The two quantities are shown to evolve to their equilibrium values.

Physically, entropy can be interpreted as a measure of the lack of knowledge (disorder) of the system. The effective temperature associated with the Bose distribution has been introduced [17] in connection with the entropy obtained in the quantum theory of relaxation for the harmonic oscillator. The entropy for an infinite coupled harmonic oscillator chain has also been calculated for classical [18] and quantum mechanical systems [19] represented by a phase space distribution function. In the present work we extract the density operator of the damped harmonic oscillator in the Lindblad theory by using a technique analogous to those applied in the description of quantum relaxation [17, 19, 20, 21]. Denoting by $\rho(t)$ the density operator of the damped harmonic oscillator in the Schrödinger picture, the von Neumann entropy $S(t)$ is given by the expectation value of the logarithmic operator $\ln\rho$:

$$S(t) = -k < \ln\rho >= -k\mathrm{Tr}(\rho\ln\rho), \tag{1.1}$$

where k is Boltzmann's constant. Accordingly, the calculation of the entropy reduces to the problem of finding the explicit form of the density operator.

The content of this paper is arranged as follows. In Sec. 2 we write the master equation for the density operator of the harmonic oscillator. In Sec. 3 we derive a closed form of the density operator satisfying the master equation based on the Lindblad dynamics. By using the explicit form of the density operator, in Sec. 4 we calculate the von Neumann entropy and time-dependent temperature and analyze their temporal behaviour. Finally, concluding remarks are given in Sec. 5.

2 Master equation for the damped quantum harmonic oscillator

The rigorous description of the dissipation in a quantum mechanical system is based on the quantum dynamical semigroups [2, 3, 8]. According to the axiomatic theory of Lindblad [8], the usual von Neumann-Liouville equation ruling the time evolution of closed quantum systems is replaced in the case of open systems by the following equation for the density operator ρ:

$$\frac{d\Phi_t(\rho)}{dt} = L(\Phi_t(\rho)). \tag{2.1}$$

Here, Φ_t denotes the dynamical semigroup describing the irreversible time evolution of the open system in the Schrödinger representation and L the infinitesimal generator of the dynamical semigroup Φ_t. Using the structural theorem of Lindblad [8], which gives the most general form of the bounded, completely dissipative Liouville operator L, we obtain the explicit form of the most general time-homogeneous quantum mechanical Markovian master equation:

$$\frac{d\rho(t)}{dt} = L(\rho(t)), \tag{2.2}$$

where

$$L(\rho(t)) = -\frac{i}{\hbar}[H,\rho(t)] + \frac{1}{2\hbar}\sum_j([V_j\rho(t),V_j^\dagger] + [V_j,\rho(t)V_j^\dagger]). \tag{2.3}$$

Here H is the Hamiltonian of the system. The operators V_j and $V_j^\dagger$ are bounded operators on the Hilbert space $\mathcal{H}$ of the Hamiltonian.

We should like to mention that the Markovian master equations found in the literature are of this form after some rearrangement of terms, even for unbounded Liouville operators. In this connection we assume that the general form of the master equation given by (2.2), (2.3) is also valid for unbounded Liouville operators.

In this paper we impose a simple condition to the operators $H, V_j, V_j^\dagger$ that they are functions of the basic observables $\hat{q}$ and $\hat{p}$ of the one-dimensional quantum mechanical system (with $[\hat{q},\hat{p}] = i\hbar I$, where I is the identity operator on $\mathcal{H}$) of such kind that the obtained model is exactly solvable. A precise version for this last condition is that linear spaces spanned by first degree (respectively second degree) noncommutative polynomials in $\hat{q}$ and $\hat{p}$ are invariant to the action of the completely dissipative mapping L. This condition implies [9] that V_j are at most first degree polynomials in $\hat{q}$ and $\hat{p}$ and H is at most a second degree polynomial in $\hat{q}$ and $\hat{p}$. Then the harmonic oscillator Hamiltonian H is chosen of the general form

$$H = H_0 + \frac{\mu}{2}(\hat{q}\hat{p}+\hat{p}\hat{q}), \quad H_0 = \frac{1}{2m}\hat{p}^2 + \frac{m\omega^2}{2}\hat{q}^2. \tag{2.4}$$

With these choices the Markovian master equation can be written [6, 10]:

$$\begin{aligned}\frac{d\rho}{dt} = &-\frac{i}{\hbar}[H_0,\rho] - \frac{i}{2\hbar}(\lambda+\mu)[\hat{q},\rho\hat{p}+\hat{p}\rho] + \frac{i}{2\hbar}(\lambda-\mu)[\hat{p},\rho\hat{q}+\hat{q}\rho] \\ &-\frac{D_{pp}}{\hbar^2}[\hat{q},[\hat{q},\rho]] - \frac{D_{qq}}{\hbar^2}[\hat{p},[\hat{p},\rho]] + \frac{D_{pq}}{\hbar^2}([\hat{q},[\hat{p},\rho]] + [\hat{p},[\hat{q},\rho]]),\end{aligned} \tag{2.5}$$

where D_{pp}, D_{qq} and D_{pq} are the diffusion coefficients and λ the friction constant. They satisfy the following fundamental constraints [6, 10]:

$$\text{i) } D_{pp} > 0, \quad \text{ii) } D_{qq} > 0, \quad \text{iii) } D_{pp}D_{qq} - D_{pq}^2 \geq \lambda^2\hbar^2/4. \tag{2.6}$$

In the particular case when the asymptotic state is a Gibbs state

$$\rho_G(\infty) = e^{-\frac{H_0}{kT}} / \mathrm{Tr} e^{-\frac{H_0}{kT}}, \tag{2.7}$$

these coefficients reduce to

$$D_{pp} = \frac{\lambda + \mu}{2} \hbar m\omega \coth \frac{\hbar\omega}{2kT}, \quad D_{qq} = \frac{\lambda - \mu}{2} \frac{\hbar}{m\omega} \coth \frac{\hbar\omega}{2kT}, \quad D_{pq} = 0, \tag{2.8}$$

where T is the temperature of the thermal bath and the fundamental constraints are satisfied only if $\lambda > |\mu|$.

3 Density operator of the damped harmonic oscillator

By introducing the real variables x_1, x_2 corresponding to the operators $\hat{q}, \hat{p}$:

$$x_1 = \sqrt{\frac{m\omega}{2\hbar}} q, \quad x_2 = \frac{1}{\sqrt{2\hbar m\omega}} p, \tag{3.1}$$

in [12, 13] we have transformed the master equation for the density operator into the following Fokker-Planck equation satisfied by the Wigner distribution function $W(x_1, x_2, t)$:

$$\frac{\partial W}{\partial t} = \sum_{i,j=1,2} A_{ij} \frac{\partial}{\partial x_i}(x_j W) + \frac{1}{2} \sum_{i,j=1,2} Q_{ij}^W \frac{\partial^2}{\partial x_i \partial x_j} W, \tag{3.2}$$

where

$$A = \begin{pmatrix} \lambda - \mu & -\omega \\ \omega & \lambda + \mu \end{pmatrix}, \quad Q^W = \frac{1}{\hbar} \begin{pmatrix} m\omega D_{qq} & D_{pq} \\ D_{pq} & D_{pp}/m\omega \end{pmatrix}. \tag{3.3}$$

Since the drift coefficients are linear in the variables x_1 and x_2 and the diffusion coefficients are constant with respect to x_1 and x_2, Eq. (3.2) describes an Ornstein-Uhlenbeck process [22]. Following the method developed by Wang and Uhlenbeck [22], we solved in ref. [13] this Fokker-Planck equation, subject to either the wave-packet type or the δ-function type of initial conditions.

In the present paper we consider the underdamped case ($|\mu| < \omega$) of the harmonic oscillator [6, 10]. If the initial condition for the Fokker-Planck equation is of a Gaussian (wave-packet) type (x_{10} and x_{20} are the initial values of x_1 and x_2 at $t = 0$, respectively)

$$W_w(x_1, x_2, 0) = \frac{1}{\pi\hbar} \exp\{-2[(x_1 - x_{10})^2 + (x_2 - x_{20})^2]\}, \tag{3.4}$$

the solution of Eq. (3.2) is given by [13]:

$$W_w(x_1,x_2)=\frac{\Omega}{\pi\hbar\omega\sqrt{-B_w}}\exp\{-\frac{1}{B_w}[\phi_w(x_1-\bar{x}_1)^2+\psi_w(x_2-\bar{x}_2)^2+\chi_w(x_1-\bar{x}_1)(x_2-\bar{x}_2)]\}, \quad (3.5)$$

where

$$B_w=g_1g_2-\frac{1}{4}g_3^2, \quad g_1=g_2^*=\frac{\mu a}{\omega}e^{2\Lambda t}+\frac{d_1}{\Lambda}(e^{2\Lambda t}-1), \quad g_3=2[e^{-2\lambda t}+\frac{d_2}{\lambda}(1-e^{-2\lambda t})], \quad (3.6)$$

$$\phi_w=g_1a^{*2}+g_2a^2-g_3, \ \psi_w=g_1+g_2-g_3, \ \chi_w=2(g_1a^*+g_2a)-g_3(a+a^*). \quad (3.7)$$

We have put $a=(\mu-i\Omega)/\omega$, $\Lambda=-\lambda-i\Omega$, $d_1=(a^2m\omega D_{qq}+2aD_{pq}+D_{pp}/m\omega)/\hbar$, $d_2=(m\omega D_{qq}+2\mu D_{pq}/\omega+D_{pp}/m\omega)/\hbar$ and $\Omega^2=\omega^2-\mu^2$. The functions $\bar{x}_1$ and $\bar{x}_2$, which are also oscillating functions, are found to be [13]:

$$\bar{x}_1=e^{-\lambda t}[(x_{10}(\cos\Omega t+\frac{\mu}{\Omega}\sin\Omega t)+x_{20}\frac{\omega}{\Omega}\sin\Omega t], \quad (3.8)$$

$$\bar{x}_2=e^{-\lambda t}[(x_{20}(\cos\Omega t-\frac{\mu}{\Omega}\sin\Omega t)-x_{10}\frac{\omega}{\Omega}\sin\Omega t]. \quad (3.9)$$

The solution (3.5) of the Fokker-Planck equation (3.2), subject to the wave-packet type of initial condition (3.4) can be written in terms of the coordinate and momentum ($<\hat{A}>=\mathrm{Tr}(\rho\hat{A})$ denotes the expectation value of an operator $\hat{A}$) as [6, 10]:

$$W(q,p)=\frac{1}{2\pi\sqrt{\delta}}\exp\{-\frac{1}{2\delta}[\phi(q-<\hat{q}>)^2+\psi(p-<\hat{p}>)^2-2\chi(q-<\hat{q}>)(p-<\hat{p}>)]\}, \quad (3.10)$$

where

$$<\hat{q}>=e^{-\lambda t}[(\cos\Omega t+\frac{\mu}{\Omega}\sin\Omega t)<\hat{q}(0)>+\frac{1}{m\Omega}\sin\Omega t<\hat{p}(0)>], \quad (3.11)$$

$$<\hat{p}>=e^{-\lambda t}[-\frac{m\omega^2}{\Omega}\sin\Omega t<\hat{q}(0)>+(\cos\Omega t-\frac{\mu}{\Omega}\sin\Omega t)<\hat{p}(0)>], \quad (3.12)$$

$$\phi\equiv\sigma_{pp}=<\hat{p}^2>-<\hat{p}>^2=-\frac{\hbar m\omega^3}{4\Omega^2}\phi_w, \quad (3.13)$$

$$\psi\equiv\sigma_{qq}=<\hat{q}^2>-<\hat{q}>^2=-\frac{\hbar\omega}{4m\Omega^2}\psi_w, \quad (3.14)$$

$$\chi\equiv\sigma_{pq}=\frac{1}{2}<\hat{q}\hat{p}+\hat{p}\hat{q}>-<\hat{q}><\hat{p}>=\frac{\hbar\omega^2}{8\Omega^2}\chi_w, \quad \delta=\phi\psi-\chi^2. \quad (3.15)$$

To obtain the explicit form of the density operator, we apply, like in [19, 21], the relation $\rho=2\pi\hbar N\{W_s(q,p)\}$, where W_s is the Wigner distribution function in the form of standard

rule of association and N is the normal ordering operator [23, 24] which acting on the function $W_s(q,p)$ moves p to the right of q. By the standard rule of association we mean the correspondence $p^m q^n \to \hat{q}^n \hat{p}^m$ between functions of two classical variables (q,p) and functions of two quantum canonical operators $(\hat{q},\hat{p})$. The Wigner distribution function (3.10), which is in the form of the Weyl rule of association [25], can be transformed into the form of standard rule of association [26] by performing the operation:

$$W_s(q,p) = \exp(\frac{1}{2}i\hbar\frac{\partial^2}{\partial p\partial q})W(q,p). \tag{3.16}$$

We get the following Wigner distribution function:

$$W_s(q,p) = \frac{1}{2\pi\sqrt{\xi}}\exp\{-\frac{1}{2\xi}[\phi(q-<\hat{q}>)^2 + \psi(p-<\hat{p}>)^2 \tag{3.17}$$

$$-2(\chi - i\frac{\hbar}{2})(q-<\hat{q}>)(p-<\hat{p}>)]\}, \tag{3.18}$$

where

$$\xi = \phi\psi - (\chi - i\frac{\hbar}{2})^2. \tag{3.19}$$

The normal ordering operator N can be applied upon the Wigner function W_s in Gaussian form by using McCoy's theorem [23, 24]:

$$N[\exp(Aq^2 + Bp^2 + Gqp)] = [J\exp(-i\hbar\gamma)]^{1/2}\exp(\alpha\hat{q}^2 + \beta\hat{p}^2 + \gamma\hat{q}\hat{p}), \tag{3.20}$$

where $\alpha = A/C,\ \beta = B/C,\ C = \sinh\Gamma/\Gamma J,\ \Gamma = -i\hbar(\gamma^2 - 4\alpha\beta)^{1/2}$, with $J = \cosh\Gamma + i\hbar\gamma\sinh\Gamma/\Gamma = 1/(1 - i\hbar G)$. After a straightforward, but lengthy calculation, we obtain the following expression of the density operator:

$$\rho = \frac{\hbar}{\sqrt{\xi}}\exp\{\frac{1}{2}\ln\frac{4\xi}{4\delta-\hbar^2} - \frac{1}{2\hbar\sqrt{\delta}}\cosh^{-1}(1+\frac{2\hbar^2}{4\delta-\hbar^2})$$
$$\times[\phi(\hat{q}-<\hat{q}>)^2 + \psi(\hat{p}-<\hat{p}>)^2 - \chi[2(\hat{q}-<\hat{q}>)(\hat{p}-<\hat{p}>) - i\hbar]]\}. \tag{3.21}$$

This form is analogous to those obtained in [19, 21]. In particular, if $D_{qq} = D_{pq} = 0$ and $\mu = \lambda$, we obtain the Jang's model [21, 27] on nuclear dynamics based on the second RPA at finite temperature. The density operator (3.20) has a Gaussian form, as expected from the initial form of the Wigner distribution function. While the Wigner distribution is expressed in terms of real variables q and p, the density operator is a function of operators $\hat{q}$ and $\hat{p}$. When time $t\to\infty$, the density operator tends to

$$\rho(\infty) = \frac{2\hbar}{\sqrt{4\sigma-\hbar^2}}\exp\{-\frac{1}{2\hbar\sqrt{\sigma}}\ln\frac{2\sqrt{\sigma}+\hbar}{2\sqrt{\sigma}-\hbar}[\sigma_{pp}(\infty)\hat{q}^2 + \sigma_{qq}(\infty)\hat{p}^2 - \sigma_{pq}(\infty)(\hat{q}\hat{p}+\hat{p}\hat{q})]\}, \tag{3.22}$$

where $\sigma = \sigma_{pp}(\infty)\sigma_{qq}(\infty) - \sigma_{pq}^2(\infty)$ and [6, 10]:

$$\sigma_{qq}(\infty) = \frac{1}{2m^2\omega^2\lambda(\lambda^2+\Omega^2)}[m^2\omega^2(2\lambda(\lambda+\mu)+\omega^2)D_{qq} + \omega^2 D_{pp} + 2m\omega^2(\lambda+\mu)D_{pq}], \tag{3.23}$$

$$\sigma_{pp}(\infty)=\frac{1}{2\lambda(\lambda^2+\Omega^2)}[m^2\omega^4D_{qq}+(2\lambda(\lambda-\mu)+\omega^2)D_{pp}-2m\omega^2(\lambda-\mu)D_{pq}], \tag{3.24}$$

$$\sigma_{pq}(\infty)=\frac{1}{2m\lambda(\lambda^2+\Omega^2)}[-m^2\omega^2(\lambda+\mu)D_{qq}+(\lambda-\mu)D_{pp}+2m(\lambda^2-\mu^2)D_{pq}]. \tag{3.25}$$

In the particular case (2.8),

$$\sigma_{qq}(\infty)=\frac{\hbar}{2m\omega}\coth\frac{\hbar\omega}{2kT},\ \ \sigma_{pp}(\infty)=\frac{\hbar m\omega}{2}\coth\frac{\hbar\omega}{2kT},\ \ \sigma_{pq}(\infty)=0 \tag{3.26}$$

and the asymptotic state is a Gibbs state (2.7):

$$\rho_G(\infty)=2\sinh\frac{\hbar\omega}{2kT}\exp\{-\frac{1}{kT}(\frac{1}{2m}\hat{p}^2+\frac{m\omega^2}{2}\hat{q}^2)\}. \tag{3.27}$$

4 Von Neumann entropy and effective temperature

By using the relations (3.13-15), we get the expectation value of $\ln\rho$:

$$<\ln\rho>=\ln\hbar-\frac{1}{2}\ln(\delta-\frac{\hbar^2}{4})-\frac{\sqrt{\delta}}{\hbar}\ln\frac{2\sqrt{\delta}+\hbar}{2\sqrt{\delta}-\hbar}. \tag{4.1}$$

Denoting $\hbar\nu=\sqrt{\delta}-\hbar/2$, we finally obtain the following expression of the von Neumann entropy:

$$S(t)=k[(\nu+1)\ln(\nu+1)-\nu\ln\nu]. \tag{4.2}$$

Since $\delta=-\frac{\hbar^2\omega^2}{4\Omega^2}B_w$, the function ν becomes

$$\nu=\frac{\omega}{2\Omega}\sqrt{-B_w}-\frac{1}{2}, \tag{4.3}$$

where

$$\begin{aligned}B_w=\exp(-4\lambda t)(2\frac{\mu}{\omega}\mathrm{Re}\frac{d_1a^*}{\Lambda}-\frac{\Omega^2}{\omega^2}+\frac{|d_1|^2}{|\Lambda|^2}-\frac{d_2^2}{\lambda^2}+2\frac{d_2}{\lambda})\\ -2\exp(-2\lambda t)[\mathrm{Re}\left((\frac{\mu}{\omega}\frac{d_1a^*}{\Lambda}+\frac{|d_1|^2}{|\Lambda|^2})\exp 2i\Omega t\right)-\frac{d_2^2}{\lambda^2}+\frac{d_2}{\lambda}]+\frac{|d_1|^2}{|\Lambda|^2}-\frac{d_2^2}{\lambda^2}.\end{aligned} \tag{4.4}$$

It is worth noting that the entropy depends only upon the variance of the Wigner distribution. When time $t\to\infty$, ν tends to $s=\omega(d_2^2/\lambda^2-|d_1|^2/(\lambda^2+\Omega^2))^{1/2}/2\Omega-1/2$ and the entropy relaxes to its equilibrium value $S(\infty)=k[(s+1)\ln(s+1)-s\ln s]$.

The expression (4.2) is analogous to those previously obtained [17, 18] in the theory of quantum oscillator relaxation and for the description of a system of collective RPA phonons [21]. It should also be noted that the expression (4.2) has the same form as the entropy of a

system of harmonic oscillators in thermal equilibrium. In the later case ν represents, of course, the average of the number operator [19]. Eq. (4.2) together with the function ν defined by (4.3), (4.4) is the desired entropy for the system. Although the expression (4.2) for the entropy has a well-known form, the function ν induces a specific behaviour of the entropy. It is clear that the time dependence of the entropy is given by the damping factors $\exp(-4\lambda t)$, $\exp(-2\lambda t)$ and the oscillating function $\exp 2i\Omega t$. The complex oscillating factor $\exp 2i\Omega t$ reduces to a function of the frequency ω, namely $\exp 2i\omega t$ for $\mu \to 0$ or if $\mu/\Omega \ll 1$ (i.e. the frequency ω is very large as compared to μ).

In the case of a thermal bath (2.7), (2.8), a time-dependent effective temperature T_e can be defined [17, 21], by noticing that when $t \to \infty$, ν tends, according to (2.24) in ref. [13], to the average thermal phonon number $< n >= (\exp(\hbar\omega/kT) - 1)^{-1}$. Thus ν can be considered as giving the time evolution of the thermal phonon number, so that we can put in this case

$$(\exp \frac{\hbar\omega}{kT_e} - 1)^{-1} = \nu. \tag{4.5}$$

The function ν vanishes at $t = 0$. From (4.5) the effective temperature T_e can be expressed as

$$T_e(t) = \frac{\hbar\omega}{k[\ln(\nu + 1) - \ln \nu]}. \tag{4.6}$$

Accordingly, we can say that at time t the system is in thermal equilibrium at temperature T_e. In terms of the effective temperature, the von Neumann entropy takes the form

$$S(t) = \frac{\hbar\omega}{T_e(\exp \frac{\hbar\omega}{kT_e} - 1)} - k \ln[1 - \exp(-\frac{\hbar\omega}{kT_e})]. \tag{4.7}$$

As t increases, the effective temperature approaches thermal equilibrium with the bath, $T_e \to T$.

5 Concluding remarks

Recently we assist to a revival of interest in quantum brownian motion as a paradigm of quantum open systems. There are many motivations. The possibility of preparing systems in macroscopic quantum states led to the problems of dissipation in tunneling and of loss of quantum coherence (decoherence). These problems are intimately related to the issue of quantum-to-classical transition. All of them point the necessity of a better understanding of open quantum systems and require the extension of the model of quantum brownian motion. Our results allow such extensions. The Lindblad theory provides a selfconsistent treatment of damping as a possible extension of quantum mechanics to open systems. In the present paper we have studied the one-dimensional harmonic oscillator with dissipation within the framework of this theory. We have first obtained the explicit form of the density operator from the master and Fokker-Planck equations. The density operator in a Gaussian form is a function of the position and momentum operators in addition to several time dependent factors. Then the density operator has been

used to calculate the von Neumann entropy and the effective temperature. The temporal behaviour of these quantities shows how they approach their equilibrium values. In a future work we plan to discuss the von Neumann entropy in association with uncertainty, decoherence and correlations of the system with its environment [28, 29].

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